M.SC. Revised Courses

REVISED

TWO YEAR FULL TIME PROGRAMME

OF

बिन्दु सं० - 12

MASTER OF SCIENCE IN MATHEMATICS (STATISTICS)

The proposed programme shall be governed by the "Department of Mathematics and Statistics, Dr. Rammanohar Lohia Avadh University Ayodhya-224001, UP.

[Effective from Academic Session 2019-20 (for new admission)]

Course Structure

M.Sc.- Previous		Theory/Internal Assessment
Semester –I		
Paper Code	**Title**	**Max Marks**
MMS-501	Advanced Real Analysis	100 (70/30)
MMS-502	Advanced Abstract Algebra	100 (70/30)
MMS-503	Topology-I	100 (70/30)
MMS-504	Probability and Statistical Methods	100 (70/30)
MMS-505	Differential Geometry of Manifolds	100(70/30)
MMS-506	Computing Fundamentals	100 (70/30)
	Total	**600**
Semester –II		
Paper Code	Title	Max Marks
MMS-601	Mathematical Methods	100 (70/30)
MMS-602	Linear Algebra	100 (70/30)
MMS-603	Topology-II	100 (70/30)
MMS-604	Applied Statistics	100 (70/30)
MMS-605	Numerical Analysis	100 (70/30)
MMS-606	Computing Application-Lab I	100
	Total	**600**
M.Sc.-Final		
Semester -III		
Paper Code	Title	Max Marks
MMS-701	Complex Analysis	100 (70/30)
MMS-702	Operations Research	100 (70/30)
MMS-703	Functional Analysis	100 (70/30)
MMS-704	Advanced Statistical Inference	100 (70/30)
MMS-705	Special Functions-I	100 (70/30)
MMS-706	Computing Application-Lab II	100
	Total	**600**
Semester –IV		

बिन्दु सं० - 12

Paper Code	Title	Max Marks
MMS-801	Ordinary and Partial Differential Equations	100(70/30)
MMS-802	Advanced Operations Research	100(70/30)
MMS-803	Graph Theory and Discrete Mathematics	100(70/30)
MMS-804	Demography	100(70/30)
MMS-805	Optional (One paper from the list)	100(70/30)
MMS-806	Viva-Voce	100
	Total	**600**
	Grand Total	**2400**

List of Optional Papers

One paper from the following list shall be opted by the student.

MMS-805(a)	:	Fuzzy Theory and its Applications
MMS-805(b)	:	Multivariate Analysis
MMS-805(c)	:	C++ and Object Oriented Programming
MMS-805(d)	:	Bio Informatics and Classifiers
MMS-805(e)	:	Special Functions-II
MMS-805(f)	:	Advanced Mathematical Modeling
MMS-805(g)	:	Riemannian manifolds
MMS-805(h)	:	Simulation

SEMESTERWISE COURSE OUTLINES (REVISED)

Paper I: Advanced Real Analysis (MMS-501)

Unit-I: Definition and existence of Riemann-Stieltjes integral, properties of the integral. Integration and differentiation, the fundamental theorem of Calculus, Integrations of vector-valued functions, Rectifiable curves.

Unit-II: Algebra of sets, outer Measure, Measurable Sets and Lebesgue measure, non-measurable sets, measurable functions.

Unit-III: The Lebesgue integration of bounded function over a set of finite measure, the integral of a non-negative functions, the general Lebesgue integral.

Unit-IV: The four derivatives, differentiation of monotone functions, functions of bounded variation, Lebesgue differentiation theorem, Differentiation of an integral, Absolute continuity.

Books Recommended:

1. W. Rudin: *Principles of Mathematical Analysis, 3rd Edition.* New Delhi: McGraw-Hill Inc., 2013.
2. H. L. Royden, and P.M. Fitzpatrick: *Real Analysis, 4th Edition.* New Delhi: Pearson, 2010.
3. G.de Barra: Measure Theory and Integration, Ellis Horwood Limited.

Paper II: Advanced Abstract Algebra (MMS-502)

Unit-I: Conjugacy relation, Normalizer, Counting principles and the class equation of a finite group, Group Theory: Series of groups, Schreier Theorem, Jordan Holder Theorem, Solvable groups, Sylow's theorems, p-Sylow subgroup.

Unit-II: Field Theory: Extensions Fields, Algebraic extension, Finite extension, Splitting fields, Algebraically closed fields, Normal extension, Separable extension, Primitive element theorem.

Unit-III: Galois group, Galois extensions, Fundamental theorem of Galois theory.

Unit-IV: Artin's theorem, Fundamental theorem of algebra (Algebraic proof).
Radical extensions, Insolvability of quintic, Constructibility.

Books Recommended:

1. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd. New Delhi, 1975.
2. V. Sahai and V. Bist, Algebra, Narosa Publishing House, 1999.
3. P. B. Bhattacharya, S. K. Jain & S. R. Noyapal – *Basic Abstract Algebra* (Cambridge).
4. T. W. Hungerford – Algebra (Springer).
5. Malik, Mordeson & Sen – *Fundamentals of Abstract Algebra (Tata MaGraw-Hill)*
6. *Sen, Ghosh & Mukhopadhyay – Topics in Abstract Algebra (University Press).*

References:

1. N.Jacobsan, Basics Algebra, Vols, I & II, W.H. FREEMAN, 1980 (also published by Hindustan Publishing Company)
2. M.Artin, Algebra , Prentice-Hall of a India, New Delhi .
3. N.S.Gopalkrishnan, University Algebra, Willy Eastern Ltd.

Paper III: Topology (MMS-503)

Unit-I: Definition and examples of topological spaces, Closed sets, Neighbourhoods, properties of Neighbourhoods and Neighbourhood systems. Closure, Interior, exterior and boundary of sets, Alternative methods of defining a topology in terms of Kuratowski's Closure operator, Interior operator, Accumulation points and derived sets.

Unit-II: Bases and Sub bases, Subspaces and relative topology, Hereditary properties, First and Second countable spaces, Dense sets, Separable spaces and their relationships.

Unit-III: Continuous functions and Sequential continuity, Open mappings, Closed mappings, Homeomorphism and topological property.

Unit-IV: Covering of sets and Sub Cover, Compact sets, Compact topological space, Finite Intersection properties, Basic and Sub basic open Covering, Countable compactness and sequential Compactness, compactness on the real line, Local Compactness, Lindelof spaces and Lindelof theorem, Compactification and One point compactification.

Books Recommended:

1. G.F.Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
2. J. L. Kelly: Topology, Van Nostrand Reinhold Co. New York, 1995.

References:

1. James R. Munkres: Topology, A first courses, Prentice Hall of India Pvt. Ltd. New Delhi,2000.
2. J. Dugundji: Topology, Allyn & Bacon, 1966 (reprinted in India by Prentice Hall of India Pvt. Ltd.)
3. K. D. Joshi: Introduction to general Topology, Willey Eastern Ltd., 1983.
4. J. Hocking & G. Young: Topology, Addision-Wesley, Reading, 1961.

Paper IV: Probability and Statistical Methods (MMS-504)

Unit-I: Random variables- discrete and continuous, distribution functions- binomial, Poisson, Exponential and normal distribution and their first four moments

Unit-II: Weak law of large numbers, Strong law of large numbers (without proof), central limit theorems, Method of least squares and curve fitting, fitting of a line, second degree parabola, exponential and Logistic curves

Unit-III: Bivariate distribution, Correlation, Regression analysis, Rank correlation, Multiple and Partial correlation with numerical problems on their applications.

Unit-IV: Definitions of random sampling, Parameter, Statistic and its sampling distribution. Expectation and standard error of sample mean. Four fundamental distributions derived from Normal viz. χ^2, t, F and Z (central).

Books Recommended

1. A.M. Goon, B.D.Gupta: An Outline of Statistical Theory, Vol. I. and Vol.
2. W. Feller: An Introduction to probability Theory, Vol. I.
3. P.G. Hoel: Introduction to Mathematical Statistics.
4. E.L. Lahmann: Testing Statistical Hypotheses.
5. C.R. Rao: Linear Statistical Inference and its Applications.
6. B.R. Bhat : Modern Probability Theory: An Introductory Text Book, 2nd Ed.
7. Goon Gupta and Das Gupt: Fundamentals of Statistics (Vol. II) & I
9. Cramme: Mathematical Methods of Statistics
10. Cochra : Sampling Techniques

Paper V: Differential Geometry of Manifolds (MMS-505)

Unit I: Definition and examples of differentiable manifolds, Tangent Spaces, Vector fields, Jacobian map, Distributions, Hypersurface of R^n.

Unit II: Standard connection on R^n, Covariant derivative, Sphere map, Weingarten map, Gauss equation, the Gauss curvature equation and Coddazi-Mainardi equations.

Unit III: Invariant view point cartan view point coordinate view point, Difference Tensor of two connections, Torsion and curvature tensors.

Unit IV: Riemannian Manifolds, Length and distance in Riemannian manifolds, Riemannian connection and curvature, Curves in Riemannian manifolds, Submanifolds.

Books Recommended:

1. NJ Hicks: Notes on Differential Geometry, D. Van Nostrand, 1965.
2. Y Matsushima: Differentiable Manifolds.
3. R.S. Mishra:: A Course in Tensors with Applications to Riemannian Geometry, Pothishala (Pvt.) Ltd., 1965.

Paper VI: Computing Fundamentals (MMS-506)

Unit I: Computer organization, number system, positional and non-positional, conversion from one system to another, computer codes and arithmetics.

Unit II: Boolean algebra, Boolean functions, logic gates and logic circuits, planning the computer programmes, algorithm and flow charting.

Unit III: Computer languages, analogy with natural languages, machine language and assembly language, high level language, compilers and interpreters, characteristics of good language, subroutines.

Unit IV: System implementation and operations, operating systems, business data processing concepts, data processing and storage hierarchy, file organization, file utilities, sorting, sending and merging, data base system,data communication and networking, transmission models.

Books Recommended:

1. P. K. Sinha: Computer Fundamentals, BPB Publication.
2. Br Eric D'Souza: Chipping-in, SKW Software Ltd.

Second Semester

Paper I: Mathematical Methods (MMS-601)

Unit I: Fourier transform, properties of Fourier transform, inversion formula, convolution, Parseval's identity, application of Fourier transforms in solving heat, wave and Laplace equation.

Unit II: Z-transforms, relation of Z-transform with Fourier transform, geometrical interpretation of Z-transform, Region of convergence, inverse Z-transform, Convolution, solving difference equations by Z-transforms.

Unit-III: Integral Equations: Integral equations of Fredholm and Volterra type, solution by successive substitution and successive approximation, integral equations with degenerate kernels. Integral equations with symmetric kernel, Eigen values and Eigen functions of integral equations.

Unit-IV: Calculus of Variations: The extremum of functionals, variation of functional, Euler equation in one and several independent variables, sufficient conditions for the extremum of a functional, variation problems with constraints, problem of geodesics and isoperimetric, Rayleigh-Ritz method of solving differential equations.

Books Recommended:

1. I. N. Sneddon: *The Use of Integral Transforms*. Mc Graw Hill, 1985.
2. R. R Goldberg: *Fourier Transforms*. Cambridge University Press, 1970.
3. L. Elsegolc: *Calculus of Variation*. Dover Publications, 2010.
4. R P. Kenwal: *Linear Integral Equation; Theory and Techniques*. Academic Press, 1971.
5. F.B. Hildebrand: *Methods of Applied Mathematics*, Dover Publications.
6. S. Pal and S.C. Bhunia: *Engineering Mathematics*. Oxford University Press, 2015.

Paper II: LINEAR ALGEBRA (MMS-602)

Unit I: Matrices: Elementary operations, reduced Row-Echelon form; consistency of system of equations, solutions of systems of equations, homogeneous system, inverse of a Matrix.

Unit II: Determinants, Cramer's Rule. Vectors, Inner Product, C-S inequality, Metric in R , triangular inequality, Linear transformations and matrices, kernel, Nullity theorem, Rank of a matrix, Similarity.

Unit III: Characteristic polynomials, Eigen values, Theorems on Eigen values and Eigen vectors. Cayley-Hamiltonian theorem. Properties of characteristic polynomials, direct sum, Jordan form and diagonalization.

Unit IV: Congruent symmetric matrices, Law of Inertia, Congruence Diagonalization of a symmetric matrix. Change of variable matrix, Change of variable and Diagonalizing. Quadratic forms, Diagonalization Alogrithms, positive definite symmetric matrices and Quadratic forms, orthogonal matrices.

Books Recommended:

1. Hoffman and Kunze, Linear Algebra, PHI, 1978
2. A.R Rao, Bhimashankaram P., Linear Algebra. (Tata Mc-Graw Hill).
3. S. Kumareson – Linear Algebra, PHI.
4. Rao & Bhimsankaran – Linear algebra, Springer

Paper III Topology II (MMS-603)

Unit-I: Separated sets, Connected set and Disconnected sets, Connected Topological spaces, Disconnected topological spaces, alternative definitions of connected and disconnected sets, connectedness on the real line, Components, Locally connected spaces and arcwise connected sets.

UnitII: Separation axioms: T_0 , T_1 and T_2- Spaces ,Regular spaces and T_3- spaces, Normal Spaces and T_4 - spaces. Urysohn's lemma and Tietze extension theorem.

Unit-III. Separation axioms: Completely Normal spaces and T_5- spaces,Completely Regular spaces and $T_{3\frac{1}{2}}$- spaces Nets and Filters, Topology and Convergence of nets. Hausdorff spaces and nets, Filters and their convergence, Ultra filters, Canonical way of converting nets to filters and Vice-Versa.

Unit-IV: Product Spaces: Product topology, Product of First countable spaces, second countable Spaces, Projection mappings, Product of Connected spaces and Compact spaces, Product of T_2.spaces. Tychonoff product topology in terms of standard subbase and its characterization, Compactness and product spaces (Tychonoff theorem).
Books Recommended:

1: G.F.Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
2: J.L.Kelly, Topology, Van Nostrand Reinhold Co. New York, 1995.

Reference Books:

3:James R. Munkres, Topology, A first courses, Prentice Hall of India Pvt. Ltd. New Delhi, 2000.
4:J.Dugundji, Topology, Allyn & Bacon, 1966 (reprinted in India by Prentice Hall of India Pvt.
Ltd.)
5:K.D.Joshi, Introduction to general Topology, Willey Eastern Ltd., 1983.

Paper IV: APPLIED STATATISTICS (MMS-604)

Unit-I: Definition and concept of Time series, Components of Time Series, Trend, Periodic Changes, Irregular Component, Analysis of Time Series, Uses of Time Series, Measurement of Trend, Graphic Method, Method of Semi Averages, Method of Curve Fitting by Principle of Least Squares Growth Curves and their Fitting, Moving Average Method, Approximation to Moving Average.

Unit-II: Index Numbers, Problems Involved in the Construction of Index Numbers, The Criteria of Good Index Number, Classification of Index Numbers, Economic Adviser's Wholesale Price Index Number, Cost of Living Index Number, Main Steps in the Construction of Cost of Living Index Numbers, the Construction of Cost of Living Index Numbers.

Unit- III: Simple random sampling, Description of stratified random sampling & its optimum allocation formulae, description of systematic sampling, cluster and two stage samplings, Analysis of variance, one-way and two-way classification.

Unit- IV: Design and experiments and their basic principles, description of design of experiments; completely randomized design, Randomized block design, Latin square design and their missing plot techniques, factorial experiments, confounding, BIBD and their properties.

Books Recommended:
1. S.C. Gupta and V.K. Kapur: Fundamentals of Applied Statistics.
2. Goon, Gupta and Das Gupta: Fundamentals of Statistics (Vol. II) & I
3. Sukhate and Sukhatme: Sampling
4. Cochran: Sampling
5. Daroga Singh: Sampling
6. Das and Giri: Design of experiments

Paper V: NUMERICAL ANALYSIS (MMS-605)

Unit I: Numerical Computation: Representation of integers and fractions, fixed point and floating point arithmetics, error propagation, loss of significance, condition and instability, computational method of error propagation.

Unit II: Polynomial Interpolation: Existence and uniqueness of interpolation polynomial, Interpolation using differences, error of the interpolating polynomial, osculatory interpolation, piecewise-polynomial approximation. Solution of Nonlinear Equations:Iterative methods, fixed point iteration, convergence of methods, polynomial equations, Miller's method, convergence acceleration.

Unit III: Solution of Linear Systems:Elimination with and without pivoting, triangular factorization, error and residual of an approximate solution. Backward errors and iterative improvement, fixed point iteration and relaxation methods.

Unit IV: Numerical Integration:Basic rules of numerical integration, Gaussian rules, composite rules, adaptive quadrature, Extrapolation to the limit, Romberg Integration. Solution of ODEs: Numerical differentiation, difference equations, Taylor series method, Euler's method and its convergence, Runge-Kutta methods, Multistep formulas, Predicator-Corrector methods, Adams-Moulton method, Stability of numerical methods, Round-off error propagation & control, shooting methods and finite difference methods for BVPs.

Books Recommended:

1. S. S. Sastry: Introductory methods of Numerical Analysis, PHI Learning Pvt.
2. E Balaguruswamy: Numerical Methods, MacMilan.
3. Numerical Analysis: Balaguruswamy

References:

1. Conte S.D. and de Boor C., Elementary Numerical Analysis - An Algorithmic Approach; 3rd edn.,McGraw Hill, 1981.
2. Henrici P.: Elements of Numerical Analysis (John Wiley & Sons, 1964).
3. Froberg C.E.: Numerical Mathematics, Theory and Computer Applications; The Benjamin Cummings Pub. Co. 1985.

Paper VI: Computer Application-Lab I (MMS-606)

Unit I: Software and its need, synchronization of software and hardware, types and its application, connection of software with firmware.

Unit II: Familiarity and hands on practice of some application software MS Word and Power Point Presentation, MS Excel and its programming.

Unit III: Theoretical and conceptual awareness and hands on practice of SPSS/Matlab/R/S/S+ softwares.

Unit IV: Problems of computing from central tendency, measures of dispersion, and properties of bivariate distribution including correlation and regression coefficients and various distributions etc.

Books Recommended:

1. S. Jain and G. Iyer: Computer Made Simple, BPB Publication, New Delhi.

2 P.K.Sinha: Computer Fundamentals, , BPB Publication, New Delhi.

3 S. Jain: Computer Organization and System Software, BPB Publication, New Delhi.

4 S Koirala: Excel, Word, Power point and Outlook, Questions and Answers, BPB Publication, New Delhi.

Third Semester

Paper I: COMPLEX VARIABLES (MMS-701)

Unit I: Function of complex variable, continuity and differentiability, Analytic functions, Cauchy Riemann equation (Cartesian and polar form). Harmonic functions, Harmonic conjugate, Construction of analytic functions. Exponential Stereographic projection and the spherical representation of the extended complex plane.

Unit II: Complex line integral, Cauchy Goursat theorem, independence of path; Cauchy's integral formulas and their consequences, Cauchy inequality, Liouville's theorem, Fundamental theorem of algebra, Morera's theorem, Maximum modulus principle, Schwarz lemma, Poisson's integral formula.

Unit III: Circle of convergence, radius of convergence. Taylor's series and Taylor's theorem, Laurent's series and Laurent theorem, Zeros and singularities of complex functions, classification of singularities: removable singularity, poles, essential singularities, Residue at a pole and at infinity, Cauchy's Residue theorem and its applications in evaluation of real integrals: integration around unit circle, integration over semi-circular contours (with and without real poles), integration around rectangular contours, Argument principle, Rouche's theorem.

Unit IV: Conformal transformations, Bilinear transformations, Critical points, Fixed points, Problems on cross-ratio and bilinear transformation.

Books Recommended:

1. L.V. Ahlfors: *Complex Analysis, 2nd Edition*. McGraw-Hill International Student Edition, 1990.

2. E.T. Copson: *An Introduction to the Theory of functions of a complex Variable*. Oxford university press,1995.
3. D. G. Zill and Shanahan: *A First Course in Complex Analysis with Applications, 2nd Edition*. New-Delhi: Jones and Bartlett.
4. A. R. Shastri: *An Introduction to Complex Analysis*. Macmillan India Ltd., 2003.
5. S. Ponnusamy and H.Silverman: *Complex Variables and Applications*. Birhkäuser, 2006.
6. R. Churchill and J.W. Brown: *Complex Variables and Applications, 6th Edition*. New- York: McGraw-Hill, 1996.

Paper II: Operations Research (MMS-702)

Unit I: Linear Programming, General Linear Programming, Simplex Method, two phase simplex method, Problem of Degeneracy, Theory of Duality, Fundamental properties of duality and theorem of duality.

Unit II: Transportation problem its formulation and solutions, Assignment problem its formulation and solutions, Differences between them, Related Problems

Unit III: Network Analysis-constraints in Network, Construction of network, Critical Path Method (CPM), PERT, PERT Calculation, Resource Analysis, Cost and Time Optimization Algorithm.

Unit IV: Dynamic Programming-recursive equation approach, Characteristics of Dynamic Programming, Computational procedure, some applications-LPP and Cargo loading problem.

Books Recommended:

1. Kanti Swarup, P.K. Gupta and Man Mohan: Operations Research, Sultanchand & sons, New Delhi.
2. C. R. Kothari: An Introduction to O.R., Vikas Publication house New Delhi.
3. J.K. Sharma: Operation Research, Theory and Applications Mac Millan India Ltd. Delhi.
4. H. A. Taha: Operational Research, Prentice – Hall of India, 1997.

Other Important References

1. F. S.Hillier, and Lieberman, G. J.: Introduction to Operations Research, 8th ed. New York: McGraw-Hill, 1990.
2. W. H.Marlow: Mathematics for Operations Research. New York: Dover, 1993.
3. R. Bronson: Schaum's Outline of Theory and Problems of Operations Research. New York: McGraw-Hill, 1982.
4. Wagner – *Principles of Operations Research* (PH)
5. Sasievir, Yaspan, Friedman – *Operations Research: Methods and Problems* (JW)
6. Schaum's Outline Series – *Operations Research*
7. Ackoff and Sieseni: Fundamental of Operations Research.

Paper III: Functional Analysis (MMS-703)

Unit- I: Banach Spaces: The definition and some examples, Continuous linear transformations, The Hahn Banach theorem.

Unit-II: The natural imbedding of N in N**, the open mapping theorem, the closed graph theorem, uniform boundedness principle, the conjugate of an operator.

Unit-III: Hilbert Spaces–The definition and some simple properties, Orthogonal complements, Orthogonal sets, The conjugate space H*, The adjoint of an operator.

Unit-IV: Self adjoint operators, Normal and Unitary operators,Projections, Finite dimensional spectral theory – spectrum of an operator, the spectral theorem.

Books Recommended:

1. A.H. Siddiqui- *Functional Analysis with applications*, TMG Publishing Co. Ltd, New Delhi
2. K.K. Jha- *Functional Analysis, Student's Friends*,1986
3. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958
4. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989
5. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994
6. G.F.Simmons, Introduction to Topology and Modern Analysis, Mc-Graw-Hill, 1963.
7. I. J. Maddox, Elements of Functional Analysis, University Book Stall, New Delhi.

Paper IV: ADVANCED STATISTICAL INFERENCE (MMS-704)

Unit-I: Estimation, Point estimation, Unbiasedness, consistency, efficiency and sufficiency of estimators. Factorization criterion. Maximum likelihood method of estimation, method of moments. Interval estimation.

Unit-II: Rao-Blackwell theorem, completeness and bounded completeness, Lehman-Sheffe theorem, Cramer-Rao bound and its generalization.

UNIT-III: Existence of U.M.V.U. estimator, its necessary and sufficient conditions, Minimum variance bound estimators. Exponential family of distributions and sufficiency.

Unit-IV: Testing of hypotheses, Critical region and most powerful test, Neyman Pearson lemma, Generalized N-P Lemma, Likelihood ratio principle, their asymptotic properties and its simple applications.

Books Recommended:

1. C.R. Rao: Linear Statistical Inference and its Applications
2. A. Goon, M. Gupta and F. A. Das Gupta: An Outline of Statistical Theory (Vol. II).
3. R.V. Hogg and A.T. Craig: Introduction to Mathematical Statistics.
4. A.Wald: Sequential Analysis.
5. A. Wald: Statistical Decision Functions.
6. V.K. Rohatgi: Probability and Mathematical Statistics, John Wiley & sons

Paper V: Special Functions -I (MMS-705)

Unit-I: Hypergeometric Functions ${}_2F_1$: Preliminaries; Definition, Convergence Conditions, Absolute Convergence, Basic Properties, The Integral representation, F[a,b;c;1] as a function of the parameters; Evaluation of F[a,b;c;1]; Gauss Theorem, Simple transformations, Relations between functions of Z and 1-Z, A quadratic transformation, Other quadratic transformations, Kummer's theorem.

Unit-II: Generalized Hypergeometric Functions (${}_pF_q$) : Definition, Convergence Conditions (without proof); The exponential and binomial functions; The Integral representation, The ${}_pF_q$ with unit argument, Saalschutz's theorem, Dixon's theorem, The Confluent Hypergeometric Functions (${}_1F_1$) : Basic properties, Kummer's first formula. Kummer's second formula.

Unit-III: Bessel Functions: Preliminaries, Definition, Bessel Differential Equation, Differential recurrence relations, A pure recurrence relation, A generating function, Bessel's integral, Index half and odd integer.

Unit-IV: Legendre's Polynomials : Definition, A generating function, Differential recurrence relations, The pure recurrence relation, Legendre's differential Equation, The Rodrigue's differential formula, Bateman's generating, functions, Hypergeometric forms of $P_n(x)$, Laplace's first integral form, Some bounds on ,$P_n(x)$, orthogonality.

Books Recommended:

1. E.D.Rainvill, Special Functions, Macmillan Co., 1971.
2. D.C.Andrews, Special functions of Mathematics for Engineers, Oxford Univ. Press, 1998.
3. N.Saran, S.D. Sharma and T.N. Dwivedi, Special Functions, Pragati Prakashan, Merrut

Paper VI: Computing Application- Lab II (MMS-706)

Unit-I: Familiarity with development and scopes of computing application in various fields including science, engineering and commerce etc.

Unit-II: Principles of object oriented programming, procedure oriented programming vs object oriented programming, basic and distinguishing concepts of object oriented programming and its advantages.

Unit-III: Beginning with C++ programming, class and structure in C++, compiling and linking, token, keywords, identifiers and constants, data types, user defined and derived data types, control structures: if else, switch, do-while, while and for statements; one and two dimensional arrays.

Unit-IV: Application of C++ for computing some selected problems of Operations Research and Numerical Analysis.

Books Recommended

1. E Balagurusamy: Object Oriented Programming with C++, Tata McGraw-Hill Publishing Company Ltd, New Delhi.
2. P.K.Sinha: New Foundation of Computing, BPB Publication, New Delhi
3. S. Jain: C++ Programming Made Simple, BPB Publication, New Delhi
4. Kanetkar: Basic Programming in C++, BPB Publication, New Delhi

Fourth Semester

Paper I: Ordinary & Partial Differential Equations (MMS-801)

Unit-I: Ordinary Differential Equations, Mathematical Models, First Order Equations, Existence – Uniqueness and continuity theorems, separation and comparison theorems, system of equations existence theorems.

Unit-II: Homogeneous linear systems, Nonhomogeneous Linear systems, Linear systems with constant coefficients.Two point boundary value problem: Introduction, The two homogeneous boundary problem, the adjoint boundary problem, the non-homogeneous boundary problem, Self- adjoint boundary problem.

Unit-III: Introduction, Basic concepts and definitions, The linear- superposition principle, some important classical linear model equations, first order p d e and its solutions.

Unit-IV: Surfaces and Curves-Classification of Ist order p.d.e. Classification of solutions-Pfaffian differential equations - Quasi-linear equations, Lagrange's method-compatible systems-Charpit's method- Jacobi's method-Integral surfaces passing through a given curve- method of characteristics for quasi-linear and non-linear p.d.e., Monge cone, characteristic strip.

Book Recommended:

1. M. D. Rai Singhania, Ordinary and Partial Differential equations, S Chand Group New Delhi.

References:

1. Tyn Myn T.U., Partial Differential Equations (Chapters 2,4,6,8)
2. John F., Partial Differential Eqautions (II Edn.) (Springer Verlag) Chapter – I
3. Weinberger H.F., Intro. to Partial Diff.Equations
4. Ian Sneddon, Elements of Partial Diff.Equations (Chapters 1,2,4)
5. Greenspan, Intro. to Partial Diff. Equations
6. Copson E.T., Classical Analysis

Paper II Advanced Operations Research (MMS-802)

Unit I

Deterministic Inventory Models, Inventory Models without Shortages and with Shortages, Probabilistic Inventory Models, Instantaneous Demand and Discrete replenishment Units, Continuous Demand and Discrete Units, Continuous Demand and Continuous Units.

Unit II

Queueing Theory, Descriptions of Queuing Systems, Classification of Queuing Models, Model (M/M/1): $(\infty / FCFS)$ and its Characteristics. Model $(M/M/1):(\infty/SIRO)$ and its Characteristics, Model (M/M/1): $(N/FCFS)$ and its Characteristics.

Unit III

Multi-server Queueing Models- (M/M/s): $(\infty/FCFS)$ and its Characteristics, Model (M/M/s): $(N/FCFS)$ and its Characteristics, Model (M/M/s): (M/GD) and its Characteristics, Model $(M/E_k/1):(\infty/FCFS)$ and its Characteristics

Unit IV: Non-Linear Programming Problems (NLPP): Formulation of a NLPP, General NLPP, Constrained optimization with equality constraint, Necessary and sufficient conditions for a general NLPP (with one constraint), with m(<n) constraints, constrained optimization with inequality constraints (Kuhn – Tucker conditions)

Books Recommended:

Operations Research: Kanti Swarup, P.K. Gupta and Man Mohan, Sultanchand & sons, N. Delhi

An Introduction to O.R.: C.R.Kothari, Vikas Publication house N.Delhi.

Operation Research: J.K. Sharma, Theory and Applications Mac Millan India Ltd. Delhi.

Taha, H.A. (1997), Operational Research, Prentice – Hall of India.

Other Important References

Hillier, F. S. and Lieberman, G. J. *Introduction to Operations Research, 8th ed.* New York: McGraw-Hill, 1990.

Marlow, W. H. *Mathematics for Operations Research.* New York: Dover, 1993.

Bazaraa, M. S.; Sherali, H. D.; and Shetty, C. M. *Nonlinear Programming: Theory and Algorithms.* New York: Wiley, 1993.

Bertsekas, D. P. *Nonlinear Programming, 2nd ed.* Cambridge, MA: Athena Scientific, 1999.

Bronson, R. *Schaum's Outline of Theory and Problems of Operations Research.* New York: McGraw-Hill, 1982.

Paper III: Discrete Mathematics and Graph Theory (MMS-803)

Unit-I: Semigroups & Monoids: Definition and examples of Semigroups and Monoids. Homomorphism of Semigroups and Monoids. Congruence relation and Quotient Semigroups. Subsemigroup and Submonoids. Direct products. Basic homomorphism theorem.

Unit II: Lattices: Lattices as partially ordered sets. Their properties. Lattices as Algebraic Systems. Sublattices. Direct products and Homomorphisms. Some Special Lattices e.g., Complete, Complemented and Distributive Lattices.

Unit-III: Boolean Algebras: Boolean Algebras as Lattices, Various Boolean Identities. The Switching Algebra example. Subalgebras. Direct Products and Homomorphisms. Join-irreducible elements, Atoms and Minterms. Boolean Forms and their Euivalence.

Unit-IV: Graph Theory: Definition of Graphs, Paths, Circuits, Cycles & Subgraphs. Induced Subgraphs. Degree of a vertex. Connectivity. Planar graphs and their properties. Trees. Euler's Formula for connected planar graphs.

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Books recommended:

1. C.L. Liu: Elements of Discrete Mathematics (Second Edition), McGraw Hill, International Edition, Computer Science Series, 1986.
2. J.P. Tremblay & R. Mnohar: Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.
3. N. Dew. Graph Theory with Application to Engineering and Computer Sciences, Prentice Hall of India.

Paper IV: DEMOGRAPHY (MMS-804)

Unit I

Source of population Data
World:Census, Registration of vital events, Demographic Surveys, Population Registers.
India: Census, civil Registration system (CRS), Sample Registration Scheme (SRS), National sample Survey (NSS), Demographic surveys and other sources.

Unit II

Source of demographic data. Scope and application of demography. Content error in demographic data. Balancing equations, Chandrasekharan-Deming formula to check completeness of registration data. Population composition and its measures. Dependence ratio.
Rates of population growth: arithmetic, geometric and exponential rates of growth; doubling time.

Unit-III

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Demographic Methods: Measurement of mortality and life table : Crude, standardised and age-specific death rate, infant mortality rates, rate by cause. Complete and abridged life table and its main features, uses of life table. Measurement of fertility : Crude birth rate, general fertility rate, age-specific birth rate, total fertility rate (TFR), gross reproduction rate (GRR) and net reproduction rate (NRR).

Unit IV

Theory of migration, types and measures of migration, migration rates. Volume of migration and its estimation. Lee's model, Zipf's model, Stowffer's model for the migration process. Hamilton's rate, Migration component, migration streams.
Reproductive Health: Concept of framework; Reproductive morbidity; prevalence of RTI (reproductive tract infection) STS's and HIV/AIDS; estimate levels and interventions.

Books recommended :

1. Keyfitz, N. (1977) Applied Mathematical Demography John Wiley & Sons N.Y.
2. Cox P.R. (1976): Demography, Cambridge University Press.
3. Spiegelman, M. (1980) Introduction to Demography Harvard University Press
4. R. Ramakumar (1986): Technical Demography, Wiley Eastern limited
5. A. Bhende & Tara Tanitkar, principle of population studies, Himalaya Publishing House Pvt. Ltd.

Paper V: Optional (MMs-805)(One paper from the list of optionals

List of Optional Papers

One paper from the following list will have to be opted by the student.

MMS-805(a): Fuzzy Theory and its Applications

MMS-805(b): Multivariate Analysis

MMS-805(c): C++ and Object Oriented Programming

MMS-805(d): Bio Informatics and Classifiers

MMS-805(e): Advanced Special Functions

MMS-805(f): Advanced Mathematical Modeling

MMS-805(g): Riemannian manifolds
MMS-805(h): Simulation

Paper VI: Viva-Voce (MMS-806)

Viva- voce shall be conducted by one internal and one external examiner as suggested by BOS on the contents of papers of IV semester.

OUTLINES OF OPTIONALS

FUZZY THEORY AND ITS APPLICATIONS

UNIT-I: Fuzzy sets, basic definition, $\propto -$ level sets, convex fuzzy sets, basic operations on fuzzy sets, Types of fuzzy sets, Cartesian product, Algebraic product.

UNIT-II:Bounded sum and difference, t –norms and t–c onorms, Extension principle, The Zadeh's extension principle.

UNIT III: Fuzzy Logic, An overview of classical logic, Multivalued logic, fuzzy propositions, Fuzzy quantifiers Linguistic variables and hedges, Inference from conditional fuzzy propositions, the compositional rule of inference.

UNIT IV: Fuzzy Expert System, concept and development, an introduction to fuzzy control, fuzzy rule base, fuzzy inference engine, Fuzzification, Defuzzifications and various defuzzificztion method (the centre of area the centre of maxima and mean of maxima methods).

Books Recommended

Fuzzy Sets and Fuzzy Logic: Theory and Applications (2015), George J. Klir / Bo Yuan (Author), Pearson

Fuzzy theory, fuzzy logic and their applications (2013), Dr A K Bhargava, S chand Company Pvt Ltd, Ram Nagar New Delhi (Based on UGC Syllabus)

Fuzzy Set Theory and its Applications, Zimmermann (2001), Springer Science+Business Media, LLC

MULTIVARIATE ANALYSIS

Unit-I: Multivariate distributions: Multinomial distribution, Multivaraite exponential distributions, Multivariate normal distribution, properties, characterizations, characteristic functions.

Unit-II: Fisher - Cochran theorem & related results. Distributions of sample functions of multivariate normal distribution. Hotlling-t^2, Wishart distribution and its properties, Mahalanobis distance.

Unit-III: Multivariate data structures, principal component & factor analysis. Path analysis, relation between sets of variables; Partial correlation, multiple correlation.

Unit-IV: Canonical correlation, Measures of distance and similarity; cluster analysis; problem of classification and discriminant analysis.

Books Recommended:

1. T.W. Anderson: Introduction to Multivariate Analysis (JW)
2. G.A.F. Seber: Multivariate Observations (John - Wiley)

3. Jonson: Multivariate analysis
4. A.M Kshirsagar.: Multivariate analysis

C++ and object Orientd Programming

Unitl I: Intoduction to Object Oriented Programming, Elements of C++, working with Functions and Structures in C++

Unit II: Classes and Objects working with Constructors and Destructors, Operator Overloading

Unit III: Inheritance working with Pointers in C++ and Polymorphism

Unit IV: Introduction to Standard Template Library

Books Recommended:

1. E Balaguruswami: C++ and object Orientd Programming.
2. Bjarne Stroustrup: *The C++ Programming Language*, Addison-Wesley, 1986 (1st edition), 1991 (2nd edition), 1997 (3rd edition), 2000 (special edition).

Bioinformatics and Classifiers

Unit I: Introduction t bioinformatics, basic elements of bioinformatics, interfacing between biology and computer application, Origin of life: an introduction, Viruses: Biology of viruses; bacteriophages, plant and animal viruses, Level of organization: prokaryotic and eukaryotic cells, multicelluarity, Five kingdom classification: salient features and outline classification, Moncra , Protista, Mycota, Plantae , Animalia.

Unit II: universal biological solvent; concepts of pH, buffer and Osmolarity, Carbohydrates: monosaccharides, oligosaccharides, polysaccharides, glycosaminoglycans, proteoglycans and glycoproteins, Lipids: fatty acids, acylglycerols; phospholipids, sphingolipids, cholesterol, and membranes; Isoprenoids, icosanoids and their biological importance.Nucleic acids: bases, nucleotides, RNA and DNA; different structural forms of DNA; different types of RNA,Proteins, Enzymes.

Unit III: Techniques of Clustering, Regression and Association, Bayesian Classifier.

Unit IV: Techniques of Decision Trees, Neural Networks with Algorithms.

Books Recommended:

Bolstad, William M. (2010) Understanding Computational Bayesian Statistics, John Wiley

ISBN 0-470-04609-8

Arthur Lesk, *Introduction to Bioinformatics*, Oxford University Press, USA,(2008).

Wallace Robert A., Sanders Gerald P., Ferl Robert J. The science of life. Publisher: New York, NY : Harper Collins, 1991. ISBN: 0673380440.
Solomon Eldra P., Berg Linda R., Martin Diana W. Biology 6th edition. Publisher: Pacific Grove, CA, Brooks/Cole Thomson Learning, 2002. ISBN: 0030335035.
Nelson David L., Cox Michale. Lehninger Principles of Biochemistry 5th Edition. Publisher: New York. W. H. Freeman. 2008. ISBN 978 0716771081.
Berg, Jeremy M, Tymoczko, John L. Stryer, Lubert. Biochemistry 6th Edition. Publisher: New York : W.H. Freeman. 2007.ISBN: 071676766X.
Hames David, Hooper Nigel. Instant Notes in Biochemistry 3rd Edition. Publisher .
Nodia, Introduction t bioinformatics, basic elements of bioinformatics, interfacing between biology and computer application.
Taylor & Francis. 2007. ISBN: 185996 2491.
Voet, Donald, Voe Judith, Pratt, Charlotte W. Fundamentals of Biochemistry: Life at the molecular Level 2nd Edition. Publisher: Asia, John Wiley & Sons. 2006. ISBN: 0471753416.
Horton, Robert, Moran, Laurence A, Scrimgeour, Perry Gray Marc, Rawn.David. Principles of biochem

Special Functions -II

Unit I: Hermite Polynomials $H_n(x)$: Definition, Recurrence relations, The Rodrigue's formula, Other generating functions, Integrals, the Hermite polynomial as a ${}_2F_0$, Orthogonality Expansion of polynomials, More generating functions, Laguerre polynomials $L_n^{\alpha}(x)$: Definition, Generating functions, Recurrence relations, The Rodrigue's formula, The Differential Equation, Orthogonality, Expansion of polynomials, Special Properties, Other generating functions.

Unit II: Jacobi polynomials, $P_n^{(\alpha,\beta)}(x)$: Definition, Bateman's generating function, The Rodrigue's formula, Orthogonality, Differential recurrence relations, the pure recurrence relation, Mixed relations.

Unit III: Gagenbauer polynomials : Definitions, Simple set of polynomials, Explicit representation, Generating function, Differential recurrence relations, Bateman's generating relation, Orthogonality,

Unit IV: The ultraspherical Polynomials : Definition, Certain Basic properties, Relation with Gagenbauer polynomials. Tchebicheff polynomials : Definition, Basic properties, Explicit Formula, Certain generating function.

Books Recommended:

1. E. D. Rainvill, Special Functions, Macmillan Co., 1971.
2. D. C. Andrews, Special functions of Mathematics for Engineers, Oxford Univ. Press,1998.
3. N. Saran, S.D. Sharma and T.N. Dwivedi, Special Functions, Pragati Prakashan, Merrut.

Advanced Mathematical Modelling

Unit I: Introduction to Biomathematics, Interaction between Mathematics and Bio-Medical Sciences, Dimensions in empirical Equation, technique of Empirical Equations, Bioarthmetic of the Human Body, Mathematics of diffusion, Solution of the diffusion equation in one dimension, Mathematical formulation of the system.

Unit II: Stability analysis, almost linear system, Stability properties and nature of critical point (0, 0). Stability of single species populations two or three species system, different type of models, linear and non linear population model: discrete time data, Stochastic model, Mathematical modeling in genetics, two species and Epidemic models.

Unit III: Matrix representation of compartment models, compartment models, Transfer coefficients, discrete transfers, continuous transfers, Solution by Eigen value analysis, Eigen value-Eigen vector solutions, solving initial value problems, computer implementation, Spectral decompositions and the power method.

Unit IV: Mathematical modeling in the Biological system, modeling of blood flow and Circulation of blood, models for diffusion of oxygen in the living tissues, models for pharmacokinetics, models for gas exchange and air flow in lungs and others.

Books Recommended:

1. S.A. Levin,Frontiers in Mathematical Biology, Spring- Verlag,1994.
2. Couston, D.R., A biological Mathematics, Edward Arnold.
3. Rubinov, S.I., Introduction to Mathematical Biology, John Wiley & Sons, New York.
4. J.N. Kapur, Mathematical modeling in Biology and medicine, East west press,1985.
5. Cullen M.R., Linear Models in Biology, Ch.1-9,
5. Andrews J.G. and Mclone R.R., (Editors) Mathematical Modelling, Ch. 11,12 - Butterworths, 1976Ellishorwood, 1985.

Riemannian manifolds

Unit-I
Riemannian metrics. Isometries. Conformal equivalence. Examples including Euclidean n-space, Spheres and Hyperbolic spaces.

Unit-II: Riemannian submanifolds. Riemannian connection. Fundamental Theorem of Riemannian Geometry via Koszul's Formula. Riemann curvature tensor. Sectional curvature. Ricci tensor. Ricci curvature. Scalar curvature. Real space forms.

Unit-III: Arc length and energy of a piecewise smooth curve. First and second variation of arc length and energy. Geodesics. Exponential map. Gauss Lemma.

Unit-IV: Complete manifolds. Hopf-Rinow Theorem. Jacobi Fields. Bonnet-Myers Theorem. Cartan-Hadamard Theorem. Cartan's Theorems (on determination of metric by curvature). Fundamental equations for Riemannian submanifolds.

Books Recommended:

1.S.S. Chern, W.H. Chen and K.S. Lam, Lectures on Differential Geometry, World Scientific, 2000.
2.MP do Carmo, Riemannian Geometry, Birkhauser, 1992.
3.N. J. Hicks, Notes on Differential Geometry, Von Nostrand, 1965.
4.P. Petersen, Riemannian Geometry, Springer 2006.
5.J. Jost, Riemannian Geometry and Geometric Analysis (6ed. Springer, 2011)

Simulation

Unit I: Modular break up of systems into subsystems with linkages, some examples. Logical and recurrence relationships, differential equation models, some examples from social and biostatistics.

Unit II: Description of a stochastic system in terms of appropriate random variable sets Generation of uniform and other specified distribution of random variables. Simulation of simple stochastic system.

Unit III:Simulation as a tool for making optimal policy choice.

Monte-Carlo methods for estimating numerical solutions of mathematical problems by random experimentation. Precision and accuracy of such solutions. Variance reduction methods.

Unit IV:Some examples of Monte-Carlo methods (i) definite integral evaluation (ii) Evaluation of individual terms of powers of a space matrix. Introduction of simulation languages and some simulation program packages.

Books Recommended:

1. Morgan : Elements of Simulation
2. Banks and Jery : Descrete Event System simulation
3. Brian D.Riplay : Statistical Simulation
4. Knuth : The Art of Computer Programming (Vol.II)
5. Bartly and Paul : Guide to Simulation